Seismic-Acoustic Sensors Topology for Interest Source Position Estimation

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Abstract:
Estimation of the positions of interest sources using seismic-acoustic sensors currently becomes more significant and important. By using the Time Direction of Arrival – TDOA method, the time delays of signal arrival from an interest source to individual sensors are carriers of information on interest source position. A typical feature of the TDOA is an ambiguous fixing of interest source position which is eliminated by statistical or other methods of signal processing. The presented contribution provides information on the found topologies of location of seismic-acoustic sensors which are characterized by suppressing or eliminating the ambiguity of fixing the interest source position, which considerably reduces computation demands on signal processing.

1. Introduction

Early in nineties, the development of new sensor systems capable of detection, identification and localization of STEALTH technology equipped targets started. Some of the sensor systems make use of the acoustic, magnetic, seismic and other physical manifestation of targets to their detection, identification and localization. This trend is caused also by the fact that exploitation of radar or optical sensors can be limited or questionable due to adverse geographic or atmospheric conditions.
One of the reasons motivating the development and testing of the new sensor systems based on non-radiolocation and non-optical principles of target detection is the fact they do not radiate any electromagnetic energy in their operation generally. Their passive principle allows them to be hidden and it gives them an advantage to be hardly detected by the reconnaissance technical means.

2. Seismic ground wave

Deformations arising in the elastic environment are of two kinds, *volumetric* and *formative*. It is possible to consider every deformation in the elastic environment as the result of the simultaneous volumetric and formative deformations. Outer force (which has the impulse character) induces time variable deformations to which movements of the environment mass particles are connected in the unconfined environment, in the area of its action. Mass particles move, they induce the tension in their neighborhood and so the deformation and tension radiate in all directions in the impulse form from the point of the excitement. At the same time, the mass particles execute the short time oscillations around their equilibrium position. The oscillations radiate from the source to the great distance, so those environment particles are passing the movement subsequently. The elastic seismic wave propagating through the environment is originated that way.
In the limited space, the surface undulation arises from the spatial waves interference. There are two basic types of the surface elastic waves – **Rayleigh waves** and **Love waves**. In case of the Rayleigh wave, particles move along elliptical trajectories in vertical planes parallel to the direction of wave propagation. The shift amplitude reaches the maximum value on the surface and it decreases exponentially with the rising distance from the surface. It dies away in the depth of several wavelengths practically.
The mechanical energy transferred by the surface wave is then concentrated in the surface layer of the thickness approximately equal to its wavelength. The way the Rayleigh wave propagates is presented on Fig.1. Love wave generates environmental oscillation in the horizontal plane upright to the direction of wave propagation. The way the Love wave propagates is presented on Fig.2.

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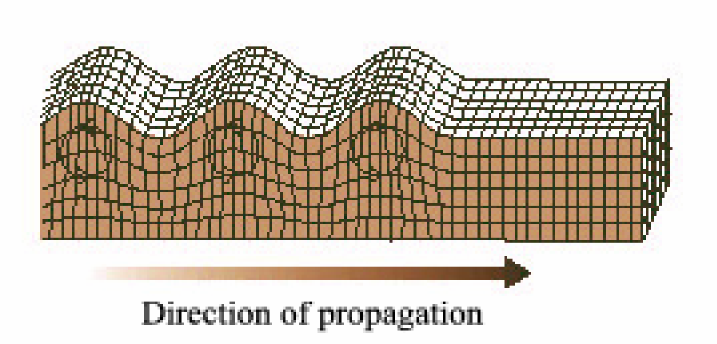

Fig.1. Propagation of the Rayleigh wave

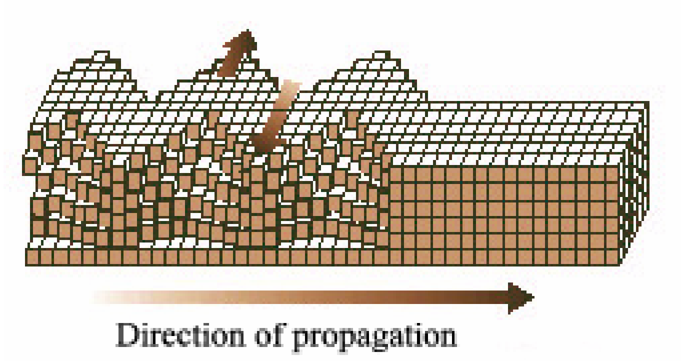

Fig.2. Propagation of the Love wave

The surface waves have the distinct polarization. Rayleigh waves are polarized in the plane upright to the Earth surface and they lead in the surface wave direction. Love waves are polarized in the horizontal plane upright to the propagation direction.

The typical feature of the surface waves in the real environment is their dispersion, i.e. dependence of the propagation speed on the period. The record of the surface waves is lengthening with the increasing distance from the epicenter owing to the dispersion and it is possible to get the sufficiently trustworthy image of the real ground movement during the surface wave propagation with assistance of the wide-band seismograph only.

Because of the different ways of the spatial and surface waves, the surface waves amplitudes decreases slower then of the spatial wave amplitudes. E.g. the spatial wave amplitudes decrease depending on the distance r as $1/r$ during wave propagation in the homogenous half-space, whereas the surface wave amplitudes decrease as $1/\sqrt{r}$. It causes that the surface waves can retain the considerably greater amplitudes then the spatial wave amplitudes in the great distances from the source.

3. Rayleigh waves coupled with airwaves

In majority of research works concerning the elastic wave propagation in the solid environment, which is in contact with the atmosphere, the atmosphere influence can be omitted because of the great difference in density. Nevertheless, the resonance coupling may occur in some cases for some frequencies, even in the cases the energy flow over the interface is low, because the transferred signal intensity can be considerable due to the forced interference.

Introduction of explosions in the air to the seismic exploration invoked investigation of the “ground scrolling” effect. It is possible to use Lamb’s theoretic conclusions which understand the acoustic wave generated e.g. by the explosion in the air as the pressure impulse propagating above the dispersion system surface. Each impulse is producing the set of waves, which are interfering together. In the end, only the waves with the phase velocity comparable with the sound propagation velocity in the atmosphere are amplified. Because the seismic wave energy is tied to the corresponding frequency, the energy propagates with the comparable phase velocity and the wave system generated by the explosion in the air consists with the group of waves with the constant frequency. The typical group velocity of the Rayleigh waves tied to the resonance frequency is approximately one half of the sound propagation velocity in the atmosphere, so the surface waves occur not until after arrival of the acoustic waves and the surface waves are registered during several periods only. This approximate description explains the side effect of the explosions made in the air, that the "ground scrolling" is part of the seismic record. The Seismic surface wave signal records invoked by the explosion of 400 grams of TNT explosive is presented for illustration in Fig. 3.

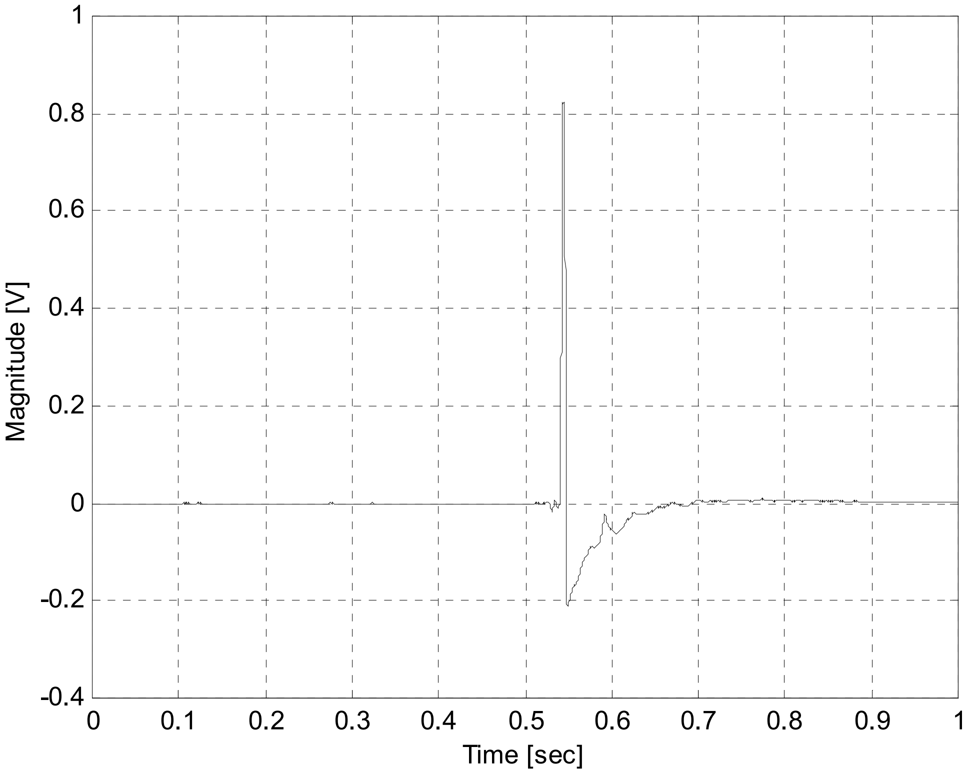

Fig.3. Time diagram of the seismic surface waves signal, explosion of 400 grams of TNT

4. TDOA (Time Direction Of Arrival) target localization method

TDOA method results from the possibility to determine the *source* position based on processing signals from stations distant from each other. A location of each station is defined. Thereat, the number of the stations has to be three at least, in the case of the static stations for determination of the source position. TDOA belongs to the complicated approaches using hyperbole properties to determine the source position. The basic parameters of the hyperbole are measurable in the time domain. As in of the triangulation approach as in this case, the assumption of synchronous operation of the particular stations has to be observed. Since, because of reasons described further, the location of the measuring stations is assumed to be relative close to each other in the source area, the synchronous operation of the stations is ensured usually in the workaday operation. Localization of he source position is based on the signal's time-space characteristics and it will be derived from estimation of the time delay of the signal arrival from the source to the defined measuring field topology.

5. Estimation of the source signal arrival time delay

The considerable effort has to be paid to finding the proper and practically applicable method of estimation of the signal arrival time delay of the source of interest. The reason is, the resulting accuracy of the source localization method depends most of all on good estimation of the arrival time delay of the source signal. It is possible to provide estimation of the time delays of the source signal arrival by means of the correlation function or the relative correlation function, respectively.

The direct method of estimation of the correlation function from the one realization in the time domain results from the acquirement of the unrestrictedly long signal. But, the source signal in our application is limited to the length of one processed segment with the finite number of the N samples. Then, it is necessary to substitute the limit calculation for approximation, i.e. by the average from the finite number of elements. If we consider that it is possible for the certain delay τ to create $N - \tau$ sample couples only, which are departed by this interval, we get the classical relation for the relative correlation function estimate:

$$R_{fg}(\tau) = \frac{1}{N-\tau} \sum_{n=1}^{N-\tau} f_{w_i}(n)\, g_{w_i}(n+\tau) \ . \qquad (1)$$

Because the estimate accuracy computed by means of the average decreases with the decreasing number of elements in the average, the accuracy of the correlation function values for the increasing τ gradually drops as well. So, it is necessary to state that we would obtain the usable reliability of the auto correlation function estimate for values $|\tau << N|$ only. So we have to analyze substantially longer source signal sequence then the argument range of the correlation function segment needed. Owing to the distinct impulse character of the source signal, the estimate of the delay represented by the shift value τ is relatively well obvious. Nevertheless, it is necessary to condition the source signal properly to gain the distinctive and utilizable value of the delay represented by the shift value τ . The approach based on application of methods characterizing the source signal envelope was indicated as suitable and utilizable.

It is possible to define the standardized envelope of the sampled signal from the source of interest, which is given by the sequence of samples *x(n)* by means of the Hilbert transformation as follows:

$$env(n) = \frac{1}{Env_{\max}} \sqrt{z^2(n) + z_H^2(n)} \quad 0 \le env\,(n) \le 1\,, \qquad (2)$$

where z_H^2 is the Hilbert transformation $z(n)$ and $Env_{\max}$ is the maximum envelope value of the signal of interest before its standardization. The following relation defines the smoothed signal envelope of the signal of interest:

$$env_W(n) = \frac{1}{Env_{W\max} W} \sum_{i=W/2}^{W/2-1} env\,(n+1) \qquad 0 \le env_W(n) \le 1\,, \qquad (3)$$

where $env(n)$ was defined by the relation (2), W is the Hann weight window length, $Env_{W\max}$ is the maximum value of $env_W(n)$ before standardization. The smoothed source signals envelope $env_W(n)$ (3) is already applicable for computation of the source signal arrival delay to the particular stations.

On the Fig.4, the diagram of the relative correlation function $R_{fg}(\tau)$ of the source signal segment using the smoothed envelope $env_W(n)$ from two stations is provided. There was distance of 15 meters between the stations. The maximum of the correlation function $R_{fg}(\tau)$ gives the delay magnitude of the source signal arrival. τ for the one sensor station against the second one given in the integral multiples of the sampling period T_{vz} of the source signal.

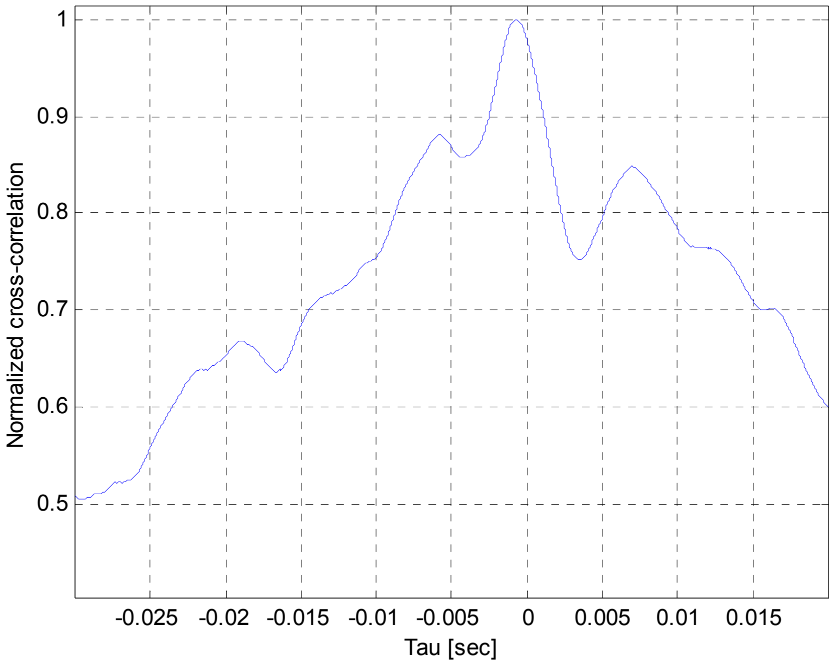

Fig.4. Diagram of the relative correlation function, τ = -77 µsec, (explosion of 400g. of TNT)

6. TDOA system

Generally, the hyperbole is the trajectory of point moving in a plane in such a way that the difference of its distances from two fixed points is constant. The interval between two adjoining hyperbolas is named spacing and it is designated by the letter *l*. Asymptotes of a hyperbole are straight lines to which hyperbole branches are closing in the infinity.
The maximum number of hyperbolas which can be constructed if the delay difference of the source signal arrival to the station *A* and *B* or *B* and *C,* respectively is expressed as the integral multiple of the shifts τ expressing the maximum functional value of the relative correlation function, is given as:

$$N_{\max} = \frac{d}{cT_{sam}}, \tag{4}$$

where: d - distance between two stations,
c - propagation velocity of the seismic surface wave,
T_{sam} - source signal sampling period.

It is necessary to specify for the source localization problem, that the system will not use one hyperbole only, but the whole system of hyperbolas. The common property of all hyperbolas will be the fact that all hyperbolas will have the identical focus, in which the particular stations *A* and *B* are situated. To determinate the hyperbola direction means to find out, where the selected hyperbole coming from the center of *A B* join points.
It is necessary to have at least one other direction and so there is the requirement for another two geographically located stations. The task can be solved by the simple exchange of indexes and it can be simplified even further in such a way that one station will be joint one for both systems. The three-position hyperbolic system with the stations *A*, *B* and *C* is created this way – see Fig.5.

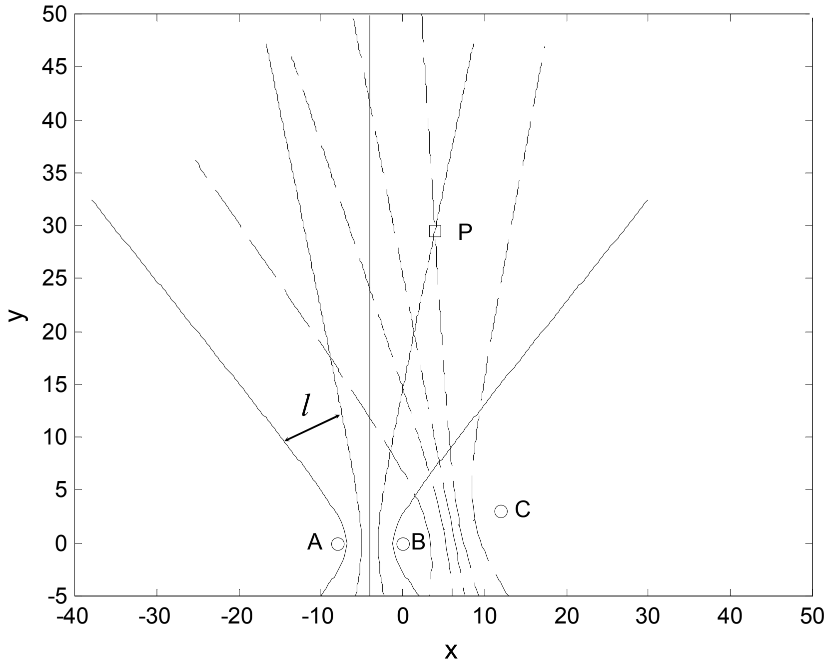

Fig.5. Localization of P point as the point of intersection of two hyperbolas

To move from the hyperbolic system to TDOA system, it is necessary to start from the fact, the source position on the point of intersection of hyperbolas can be determined by measuring time. It is possible to come out from the presumption that the source signal is reaching the particular stations in definite time intervals. But, they are not known in advance because the position of the source of interest is not known. The analysis can be done as in Fig.6.
Generally:

$$\tau_{AB} = t_a - t_b \,, \tag{5}$$

$$\tau_{BC} = t_b - t_c \,. \tag{6}$$

It is necessary to determine quantities τ_{AB}, τ_{BC} with the high accuracy, because they carry the information on the source position. Each of them expresses one of the source position coordinates in the hyperbolic coordinate system.

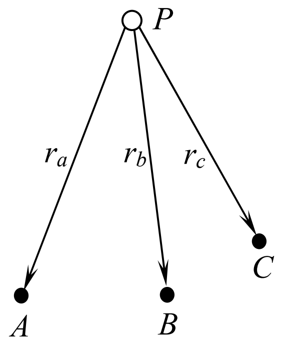

Fig.6. The basic diagram of the source signal flow between the source and TDOA stations

7. Hyperbole approximation by its asymptote

We depart from the classical TDOA model herein already. There are reasons distinguishing the classical TDOA model from the system based on processing Rayleigh waves coupled with airwaves:

- The propagation velocity of man-made seismic surface waves coupled with airwaves is equal to the sound propagation velocity in the air,
- The difference of delay in the source signal arrival to particular stations is expressed as the integral multiple of the source signal sampling period,
- The distance between the particular stations has to be relatively small, some tens of meters practically, to estimate values of relative delays of the source signal arrival to the particular stations by application of the relative correlation functions.

It was verified, if it is possible to approximate hyperbolas by their asymptotes. On Fig.7. there are diagrams of hyperbolas corresponding to the delays $25T_{sam}$, $50T_{sam}$, $75T_{sam}$ and $90T_{sam}$ ($T_{sam} = 250\mu\sec$ is chosen for illustration), and their corresponding asymptotes. Distance between *A* and *B* stations is 8 meters.

Distance of asymptotes from hyperbolas depending on the distance from the stations is stated in Fig.8. It is obvious, that the approximation error depends on the distance from the station and at the same time is affected by the delay parameter as well. But, the approximation error goes down very quickly and it is less then 15 cm for 40-meter distance from the station.

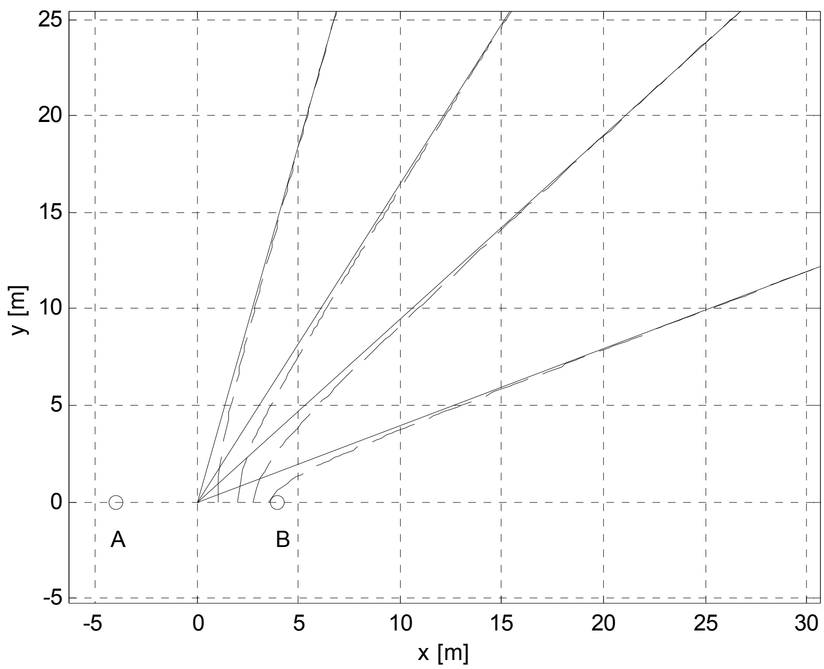

Fig.7. Diagram of hyperbolas and corresponding asymptotes

With regard to errors arising at estimating the delay of the source signal arrival to the particular stations, it is possible to omit the approximation error when approximating a hyperbole by the asymptote. In the further solution of the source localization problem, it is possible to work with asymptotes of the particular stations only and it is not necessary to solve the intersection of two or more hyperbolas, but two or three straight lines – asymptotes.

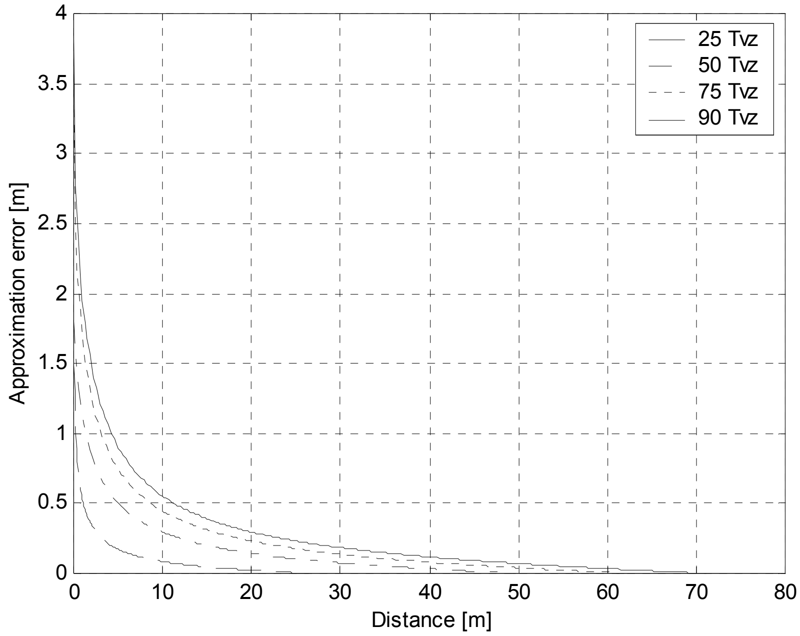

Fig.8. Approximation error for approximation of a hyperbole by the asymptote

8. Localization accuracy

At the general evaluation of the sensor systems feasibility, the term "surface of indeterminateness" S_N is used for expressing the surface element on which a source is randomly situated. The surface of indeterminateness expresses the boundary accuracy of the sensor system. It is not possible to increase this accuracy from the point of view of the measurement principle and in a way it is a measure of the surface resolution. Evaluation of the sensor system accuracy directly in the Cartesian coordinates can be provided by hyperbola approximation with asymptotes even if the source position is generally defined by its hyperbolic coordinates.

Nevertheless, it is obvious from the preceding paragraphs, the geometric factors of the stations locations together with the used signal sampling period for the source of interest influence the resulting localization accuracy for the source of interest. Some other non-geometric factors are not insignificant as well.

Up to now, without loss of the commonness, we suppose there are two hyperbolic systems. The first with the *AB* stations and the second with the *BC* stations, where the point *B* is common for both systems. We suppose both systems are situated on one straight line and we do not consider existence of indetermination of the source localization in this moment.

The source position is defined by the projection of two asymptotes a_A, a_C on the axis x and y, where the parametric expression of asymptotes is given by the estimated delay values of the signal arriving from the particular hyperbolic systems. We consider expression of the particular delay values in discrete values of the sampling period.

Points of intersection of two successive asymptotes propagating with the coefficient l_a with two successive asymptotes propagating with the coefficient l_c form a tetragon with vertexes $\{P_1,P_2,P_3,P_4\}$ representing the surface of indeterminateness in the source detection. Then, the distance of projections h_x between points P_3, P_4 on axis x a h_y and between points P_3,P_4 on axis y gives the sensor system accuracy relative to values of the parameters l_a , l_c., which are function of the distance of the point P from other stations, their spacing and the signal sampling period of the source of interest, as mentioned above.

The localization error δ_x and δ_y can be written as:

$$\delta_x = \pm\left(\frac{x_{P4} - x_{p3}}{2}\right), \tag{7}$$

$$\delta_y = \pm\left(\frac{y_{P4} - y_{p3}}{2}\right). \tag{8}$$

The size of the surface of indeterminateness formed by the tetragon with the vertexes P_1,P_2,P_3,P_4 and where each vertex is defined in the Cartesian coordinates can be written as:

$$S_N = \frac{1}{2}\left|\begin{vmatrix} x_{P1} & x_{P2} \\ y_{P1} & y_{P2} \end{vmatrix} + \begin{vmatrix} x_{P2} & x_{P3} \\ y_{P2} & y_{P3} \end{vmatrix} + \begin{vmatrix} x_{P3} & x_{P4} \\ y_{P3} & y_{P4} \end{vmatrix} + \begin{vmatrix} x_{P4} & x_{P1} \\ y_{P4} & y_{P1} \end{vmatrix}\right|. \tag{9}$$

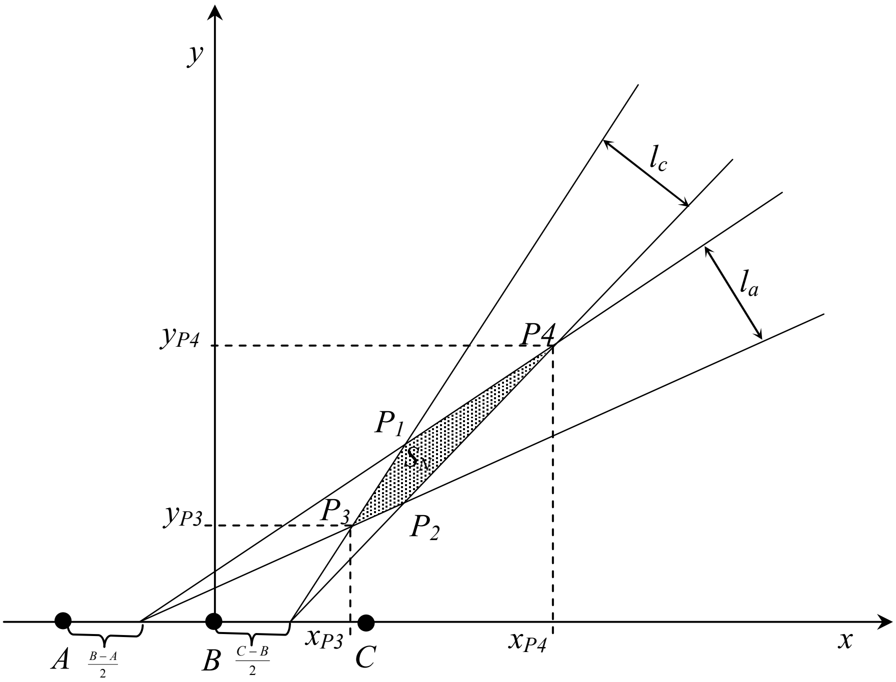

Fig.9. Representation of the source localization accuracy

9. Estimation of the source position

The source position estimate is defined as the intersection of asymptotes belonging to two systems, which have one common station. In our case, this common station was designated by the point *B* and the source position is determined from the acquirement of τ_{AB} and τ_{BC} . Nevertheless, there can exist not one but also two points of intersection defining the source position as the result ensuing from the very substance of the TDOA system.

The possible way out from this situation is to locate particular stations so, that one of them is not located on the straight line connecting other two. However even in this case, two intersections may exist, if the particular delay values τ_{AB} and τ_{BC} would have either respectively different signs or one delay τ_{AB} a τ_{BC} would be of the zero value. This situation is illustrated in Fig.10. The intersection corresponding to the correct source position estimate is designated with the symbol □ , the false intersection point is designated with the symbol *.

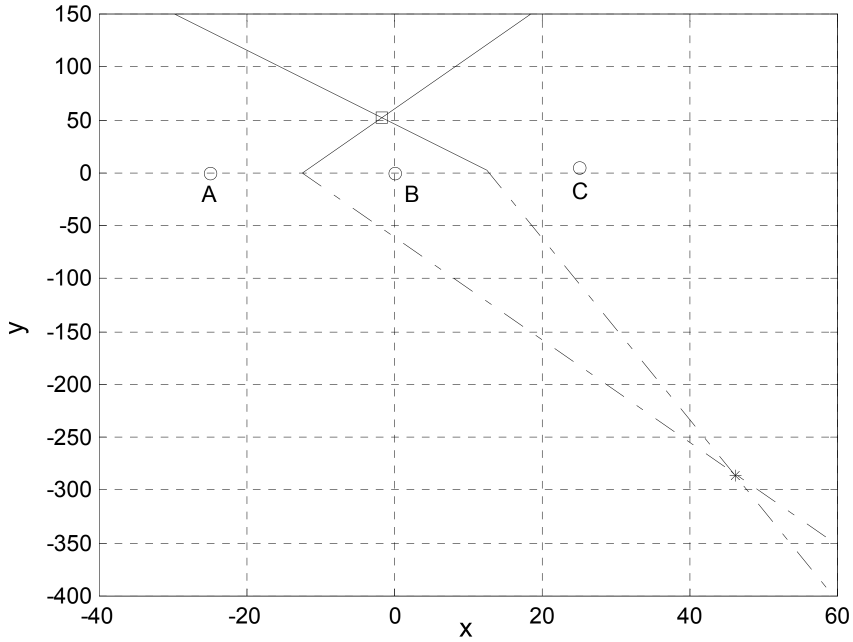

Fig.10. Creation of two intersection points given by the delay τ_{AB} a τ_{BC}

So up to now, we solved the source position estimate from three stations and two delays τ_{AB} and τ_{BC} . But, there is the variant to use four stations *A*, *B*, *C* and *D* and the corresponding delay values of the source signal arrival: τ_{AB} , τ_{BC} , τ_{CD} .

If we find the proper topology, only the three intersection points will determine the correct source position and the false position will be determined by maximum two or less intersection points. The case of the properly chosen topology is illustrated in Fig.11. The intersection points corresponding to the correct source position estimate are designated with the symbol □ , the false intersection point is designated with the symbol *.

Then for example, to estimate the source location, methods of the cluster analysis can be successfully applied. E.g., we obtain two cluster centroids using the Forgy's approach with the constant number of clusters. One cluster centroid determines the correct source position estimate and it will possess the defined pertinence function to three asymptote intersection points. The second cluster centroid determining the false source position estimate will possess the defined pertinence function to one or two intersection points. Furthermore, the functional of the sum of the squares of deviations from the decomposition cluster centroids usually has high values for this case.

Hereby this simple criterion – it is easy to distinguish the correct value from the false value of the source location by comparing two magnitudes of the functional of the sum of squares of the deviations. On Fig.12, there is an example of source position determination expressed by the centroid of the cluster T.

On next diagrams, the kinematics error estimates in areas of unambiguous source location of some suitable topologies are represented, for the area of indeterminate point corresponding to value of 50 m^2 (blue), 100 m^2 (green), 150 m^2 (brown). Presented diagrams were estimated using the sampling frequency f_{sam} = 8kHz. No real atmospheric influences on the propagation velocity of sound waves were considered. The values of the station locations *A*, *B*, *C* and *D* are stated in the Cartesian coordinates in meters.

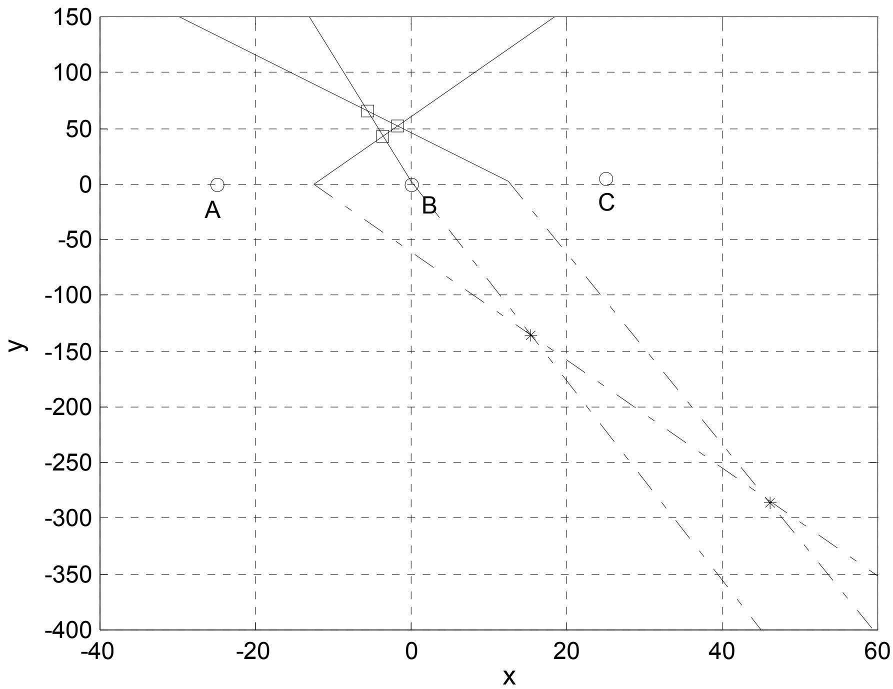

Fig.11. Properly chosen topology of the station location

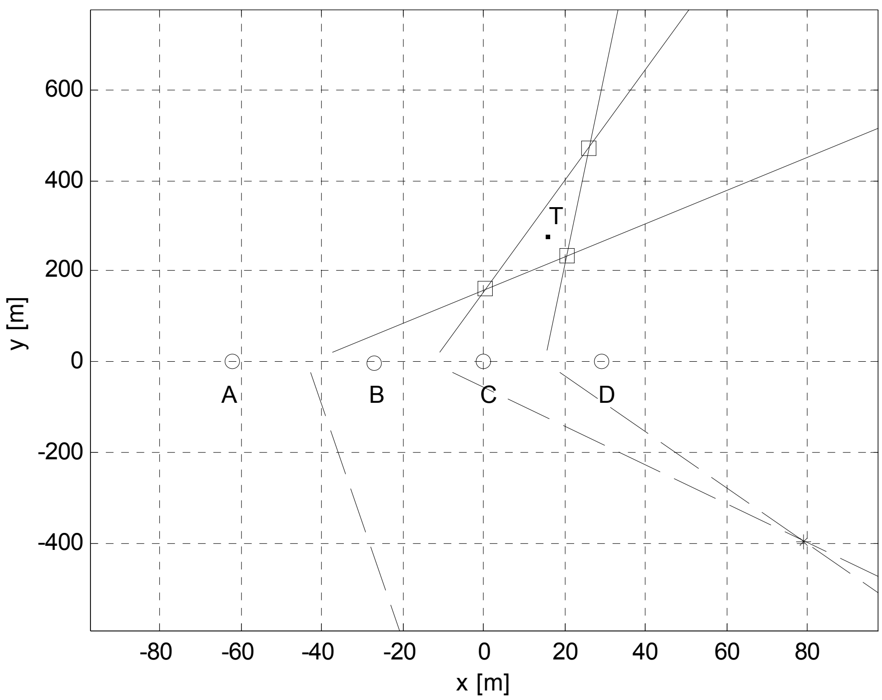

Fig.12. Illustration of the source position estimate as expression of the cluster T centroid. The particular centroids are represented with dots in the diagram.

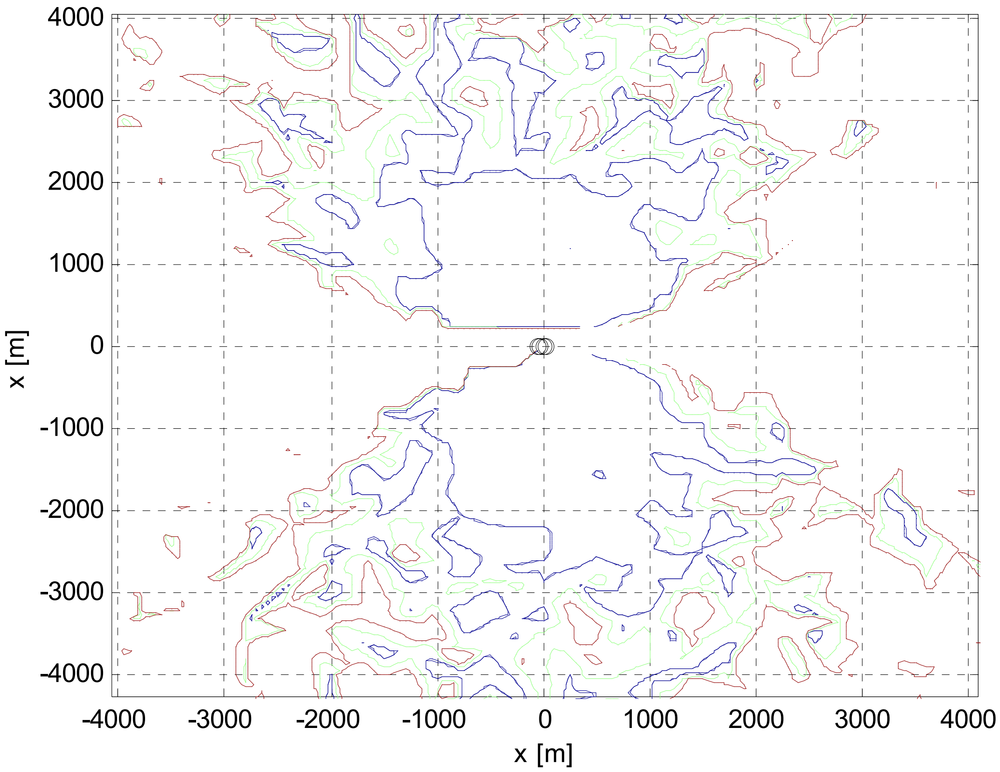

Fig.13. The error estimate in the area of unambiguous determination of the source, the topology no. 1

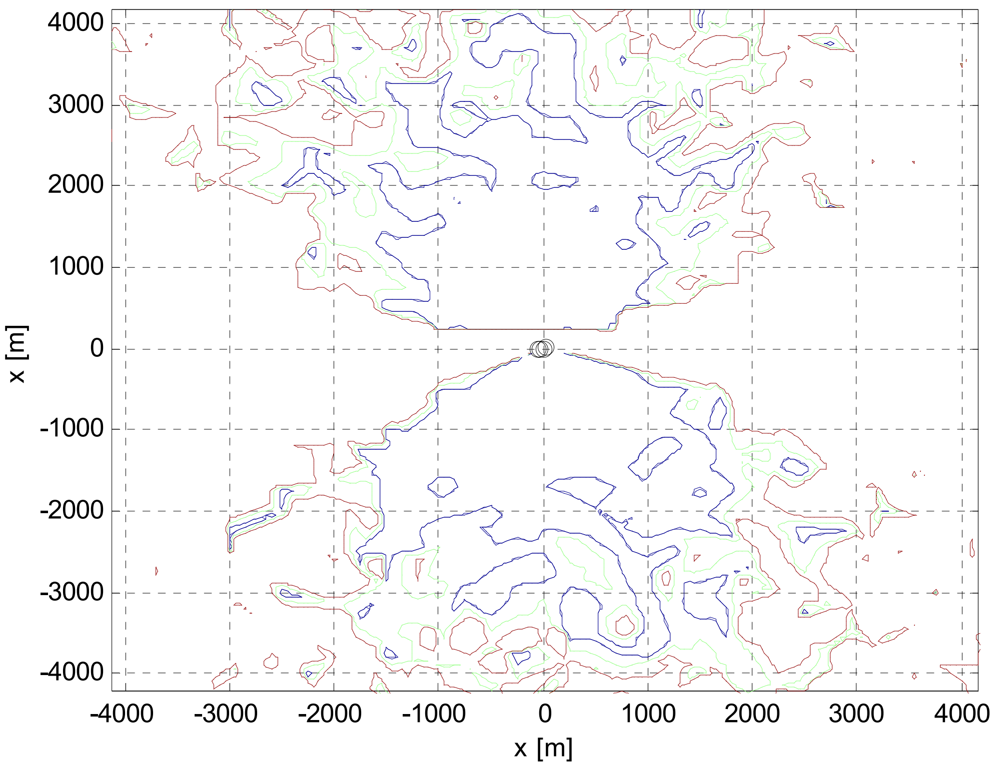

Fig.14. The error estimate in the area of unambiguous determination of the source, the topology no. 2

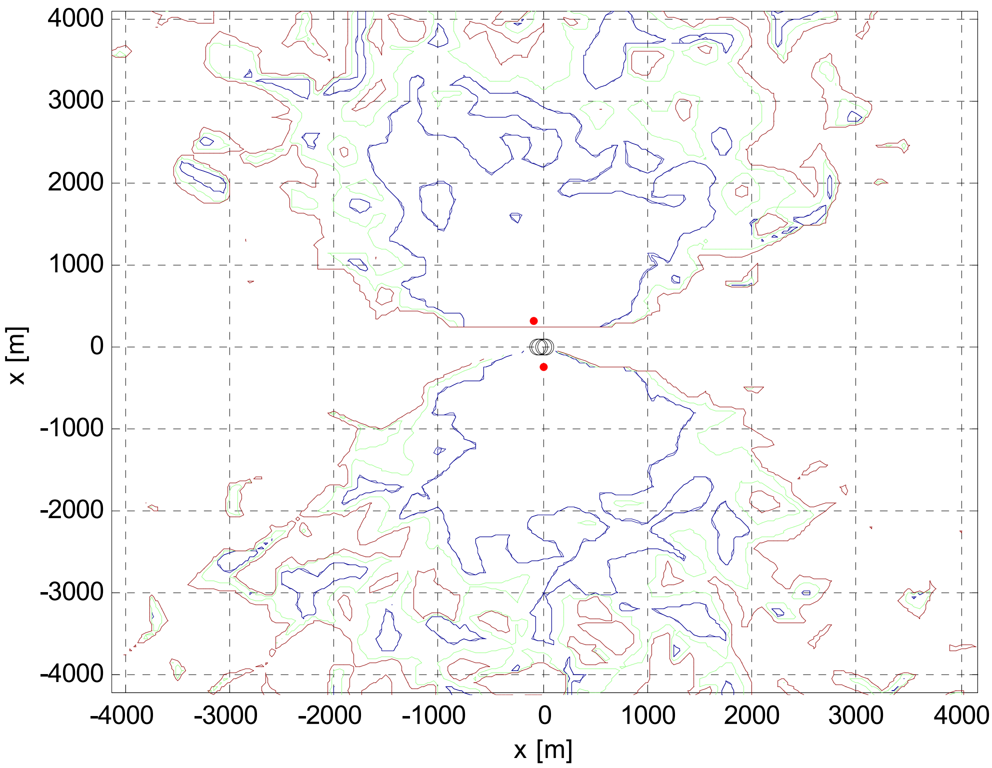

Fig.15. The error estimate in the area of unambiguous determination of the source, the topology no. 3

10. Conclusion

It is possible to state here, the source localization problems are substantially more demanding both in time and computation and less mapped then the detection problems or source identification, for example. Theoretic conclusions and practical experience proved, that the seismic surface waves can be used to estimate the source position if observing some fundamental rules and presumptions, which can be summarized into following articles:

- The critical point in estimating the source location is the right estimation of the time delays of the source signal arrival to the particular stations and acquirement of the velocity of Rayleigh waves propagation coupled with the airwaves.
- It is possible to get satisfactory results of the source position estimate, the lower values will exhibit lower estimation errors.
- It is possible to eliminate the false source position using all three delay values of the source signal arrival and under presumption of the suitable topology of station location.
- The accuracy of the source position estimate is increasing with the rising distance between particular stations, but the verified distance between stations was approximately 50 meters, when it was still possible to estimate source signal arrival time delays.
- It is possible to omit the estimation error of the source position caused by inaccurate positioning of the particular stations in the interval of values ± 0.4 cm.

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